The Elastic Hysteresis of Steel.

By Bertram Hopkinson, F.R.S., and G. Trevor Williams.

(Received October 16,—Read November 21, 1912.)

In a recent communication to the Society* one of us described a machine whereby a bar of steel 4 inches long by $\frac{1}{4}$ inch diameter can be submitted to direct alternating stress at the rate of 120 cycles per second or more. The machine is worked by the pull of an electromagnet excited by alternating current, the pull being magnified from 20 to 60 times by resonance between its period and that of a weight attached to one end of the piece, which behaves as a spring. The stress varies between equal limits of tension and compression, and may be of any desired range up to 30 tons per square inch or more. The piece is fitted with an optical extensometer by which the extreme change of length of the piece in a cycle can be observed while the machine is in action and the range of stress calculated. An independent measure of the stress can be obtained by observation with a microscope of the movement of the weight attached to the end of the piece, whose acceleration is the chief element determining the tension or compression. Full details of these measurements are given in the paper referred to, and it will suffice to state here that similar precautions to secure accuracy in the measurements of stress were taken in the course of the work to be described, and that from the agreement between the different methods it may be taken as certain that these measurements are correct to about half a ton per square inch.

The present paper contains an account of experiments which we have made with the alternating stress machine with the object of measuring the energy dissipated by elastic hysteresis when steel undergoes cyclical variations of stress within the elastic limit. The method used is to measure the fall of temperature between the centre and ends of the test piece when it is undergoing continuous alternating stress through a constant range. The fall of temperature is proportional to the rate at which heat is being generated and conducted away, and the absolute rate of dissipation in ergs per cubic centimetre can readily be obtained by passing an electric current along the specimen when at rest, and finding the relation between the temperature and the energy dissipated by resistance.

The steel used was the same material as in the previous experiments on fatigue. It contains about 0·18 per cent. carbon and 0·7 manganese, and it breaks under a load of about 29·5 tons per square inch (46·5 kgrm. per

* 'Roy. Soc. Proc.,' 1912, A, vol. 86, p. 131.

square millimetre) with an elongation of 16 per cent. (on 8 inches). According to the fatigue test made by Dr. Stanton at the National Physical Laboratory, the limiting range of stress is about 25 tons per square inch (say, 40 kgrm. per square millimetre). At the much higher speed of reversal reached in our machine the resistance to alternating stress is considerably greater, and more than one piece has remained unbroken after ten million cycles or more between the limits of $14\frac{1}{2}$ tons tension and $14\frac{1}{2}$ tons compression, giving a total range of 29 tons.

The test-piece is shown in fig. 1. A detailed drawing of the extenso-

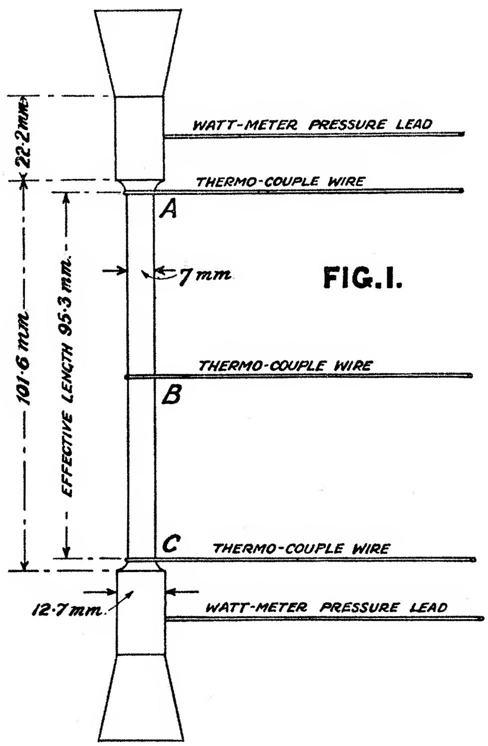


FIG. I.

meter is given in fig. 3 of the previous paper. For measuring the temperature three constantan wires are fixed to the reduced portion, one at the middle and one at each end (points A, B, and C in the figure). The middle wire and one of the end wires are connected to a galvanometer whose deflection is nearly proportional to the difference of the temperature between them. By using three wires and taking the mean of the falls of temperature in the upper and lower halves of the piece any flow of heat from external causes, such as arises for example from the eddy currents induced by the attracting magnet in the weight and in its attachments, is eliminated.

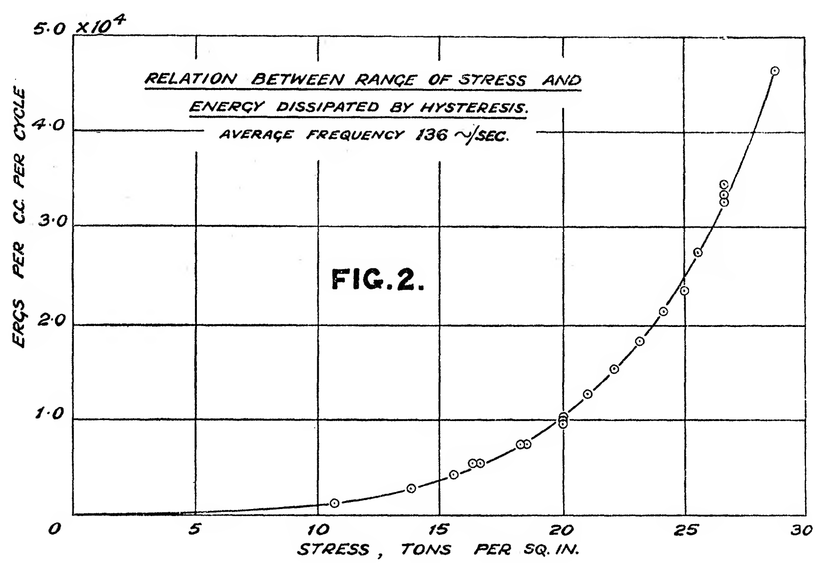
The arrangements were such that the temperature difference between the centre and the end could be measured correct to about 1/20° C. Under a stress of 28 tons range the fall of temperature is about 10° C.

The determination of the dissipation of energy corresponding to a given fall of temperature is effected by passing alternating current along the piece, which is kept in position in the machine, only the extensometer being removed. The current is taken through the fixed coils of an ordinary suspended coil watt-meter, the leads of whose shunt or pressure coil are attached near the ends of the reduced portion of the test-piece (fig. 1). The watt-meter measures the energy dissipated between the points at which these leads are attached; that dissipated in the reduced part between the points A and C is obtained by multiplying the measured watts by 0·85.* Simultaneous readings of the watt-meter and of the thermo-couple galvanometer are taken and a direct calibration of the latter is thus obtained in ergs per cubic centimetre. This is all that is required for the purpose of these experiments; the readings of the thermo-couple galvanometer were, however, also calibrated in terms of temperature, and it was found that the dissipation of energy shown on the watt-meter was accurately proportional to the temperature difference. The apparent thermal conductivity of the steel calculated from these observations, on the assumption that the whole of the heat is removed by conduction along the metal, was 0·17. The true conductivity of this material is probably about 0·14, whence it may be inferred that about five-sixths of the heat is in fact removed by conduction, the remainder escaping to the air surrounding the rod.

In making an experiment on elastic hysteresis, the machine was set to work at a certain stress, which was kept constant. The temperature rose for about five minutes and then remained steady. The experiment was usually continued for from half an hour to one hour, continuous observation being kept of the temperature and of the stress. The results are collected in fig. 2. The actual observations are shown, and it will be seen that fairly consistent values have been obtained. It has been found possible to repeat the measurement of hysteresis loss at the higher stresses within 2 or 3 per cent., though the different measurements were separated by considerable intervals of time, during which the apparatus was taken to pieces or disturbed in various ways.

* This factor was obtained by a special measurement in which watt-meter leads were attached at A and C, and the reading compared with that shown (for the same current) when the leads were attached at the outer points. It agrees with the result of calculation on the assumption that the alternating current is confined mainly to the skin of the metal, so that the effective resistances of different portions are inversely proportional to their diameters.

The energy dissipated in hysteresis increases about as the fourth power of the stress range. It is interesting to note that under a range of stress of, say, 25 tons per square inch (39 kgrm. per square millimetre) the energy dissipated per cycle by elastic hysteresis (25,000 ergs per cubic centimetre per cycle) is of the same order of magnitude as that dissipated by the magnetic hysteresis of similar material in fairly strong magnetic fields. There seems to be no reason to suppose that in either case is there any cumulative change in the properties of the material associated with the work which is being done upon it. But if the stress range be increased, the point must come at which there is such a cumulative effect, resulting ultimately in the destruction of the material by fatigue. The first sign of such a change would



probably be an increase in the energy dissipated by hysteresis. The biggest stress range in these experiments was 28·6 tons per square inch (45 kgrm. per square millimetre) and there was no sign of any increase in hysteresis after half an hour of the application of this range of stress, corresponding to about one quarter of a million reversals. From Dr. Stanton's results, however, it would appear most probable that under this range of stress, if applied 20 times per second, the material would break after less than 100,000 reversals, and that therefore there must be *some* permanent effect, though it is perhaps almost negligibly small at the higher speed of reversal.

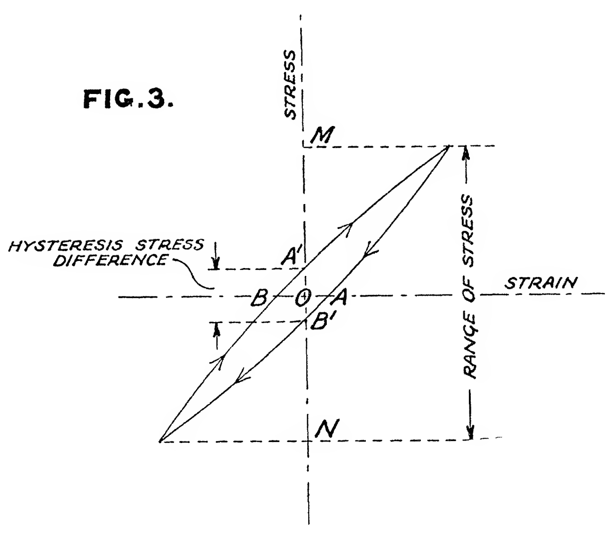
The experiments are being continued with the object of finding evidence of this increased hysteresis and generally of discovering how the hysteresis loss is related to the elastic limit.

Static Tests.

The effect of elastic hysteresis is that the stress-strain diagram corresponding to a cycle ranging between equal tension and compression is a closed loop as shown in fig. 3, instead of a straight line.* It is the work represented by the area of this loop which is measured in the experiments which have been described. It appeared important to get some approximation to the stress-strain curve obtained in a static test, in order to ascertain whether the speed of reversal had any effect on elastic hysteresis.

The only absolute measurements of elastic hysteresis of which we are aware are those described by Ewing, who loaded and unloaded a long wire by means of weights.† Traces of apparent hysteresis have been observed in ordinary measurements of elasticity with an extensometer and testing machine, but there is always a doubt in such cases whether a large part of the difference observed between loading and unloading is not due to friction in the testing machine or in the extensometer. The difference of length to be measured is not more than one hundredth of the total variation of strain, and on a piece 4 inches long amounts to but 1/50000 inch, so that the measurement is a very delicate one, an error of one-millionth of an inch in absolute length being equivalent to 5 per cent. in the result.

FIG. 3.



In the present instance no attempt was made to determine a complete stress-strain curve, but the maximum difference of length AB (fig. 3) was

* The width of the loop is much exaggerated.
† 'Brit. Assoc. Report,' 1889, p. 502.

measured. The same piece of steel as that used in the alternating stress machine was first subjected in a testing machine to compressive stress of the amount desired, say, 10 tons per square inch. It was then removed from the compression machine and loaded in tension with weights and a lever to 10 tons per square inch. After this process of alternate compression and tension had been repeated several times in order to get the piece into a cyclical state, measurements of length were taken by means of an extensometer, first, after the piece had been put into the testing machine, having just previously been loaded to 10 tons per square inch compression, and, second, after a load of 10 tons per square inch tension had been applied and removed. The piece was a little longer in the second case than in the first, and the difference in length is the quantity sought. The advantage of this procedure is that there is no measurement of load; when taking both readings the piece was hanging practically free, and there was no possibility of any stress in it. The only errors are those due to strain and backlash in the extensometer, and it was found possible to eliminate these. The resulting values of A′B′ (the stress difference corresponding to the change of length AB) are probably correct within 1/100 ton per square inch.

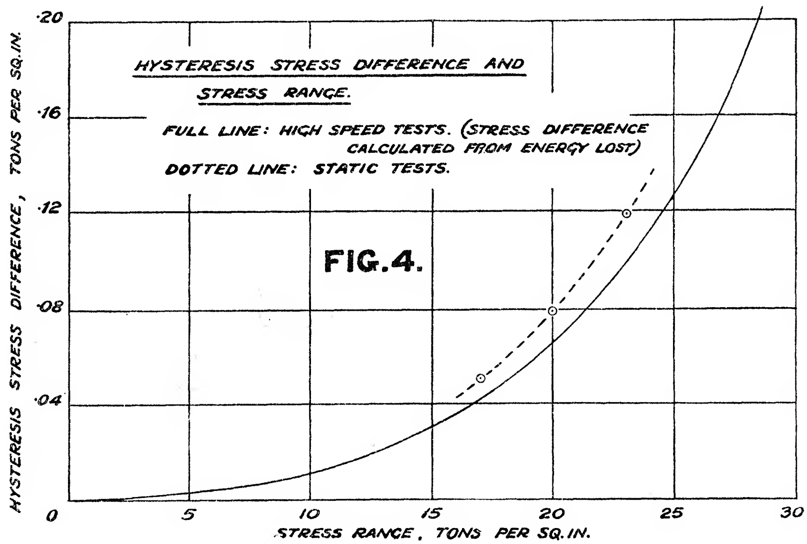
Details of the measurements and of the extensometer are given in an Appendix. The following are the final results. In each case the cycle of loading was between equal limits of tension and compression, and the "range of stress" given in the first column is twice either of these limits:—

Range of stress (tons per sq. in.)	17·0	20·0	23·0
Corresponding stress difference (A′B′) (tons per sq. in.)	0·05	0·08	0·12
Length difference (AB) (millionths of an inch)	15	24	36

These figures agree as regards order of magnitude with those given by Ewing. The range of stress in one of his experiments was about 15 tons per square inch, the limits being roughly 5 tons and 20 tons per square inch respectively (both in tension). At the intermediate load of $12\frac{1}{2}$ tons the strain in the wire was greater during the unloading part of the cycle than that corresponding to the same stress when loading, by about 1/150 part. At 17 tons range in the above table the corresponding ratio is 0·05/8·5, or 1/170.

The area of the hysteresis loop can, of course, only be roughly guessed. Assuming, as is probable, that the loop is of the lenticular form shown, its area must lie between AB . MN and half that amount. If the two sides are arcs of circles the area is $\frac{2}{3}$ AB . MN. On the assumption that this is, in

fact, the area, the value of AB has been calculated from the work done per cycle as measured in the high speed alternating stress machine. The width of the loop on the stress line A′B′ is AB multiplied by E, and this has been plotted in fig. 4. On the same figure is shown the value of A′B′ as determined from the statical experiments.



It will be observed that the stress difference calculated from the energy loss bears a substantially constant ratio of 0·8 to that measured statically. This ratio, of course, depends entirely on the factor chosen for calculating the area of the loop from its dimensions. On the extreme suppositions that this factor is 1 and $\frac{1}{2}$ respectively (instead of $\frac{2}{3}$), the ratios of high speed to static hysteresis would become 0·53 and 1·06 respectively. It seems probable, therefore, that the hysteresis in cycles performed 120 times per second is (if anything) less than that found in static tests, but it is unlikely that the difference is more than 30 per cent. Lord Kelvin, as the result of his experiments on the torsional oscillations of wires, was led to no very definite conclusion as to the relation between the dissipation of energy, in a vibration of given amplitude, and the period, but apparently thought that the loss increased slightly with the speed.* Wire, and especially the outer skin of wire (which is alone operative in torsional experiments), is, however, in an abnormal condition, and may give results which are both irregular (as was found by Kelvin) and different from those found in a bar of the same material.

* ‘Math. and Phys. Papers,’ vol. 3, p. 24.

Appendix.

The extensometer is shown diagrammatically in fig. 5. It is similar in principle to the well-known Ewing instrument, except that a tilting mirror observed from a distance through a telescope is substituted for the microscope. Two rings, A and B, surround the thickened part of the test-piece, and each is fixed to it by a pair of pointed screws so that the rings pivot about the axes C and D respectively, which are fixed in the piece. The rod E is rigidly fixed at its lower end to the ring B, and its rounded upper end engages with a conical recess in the end of a screw fixed in A, being held up by the

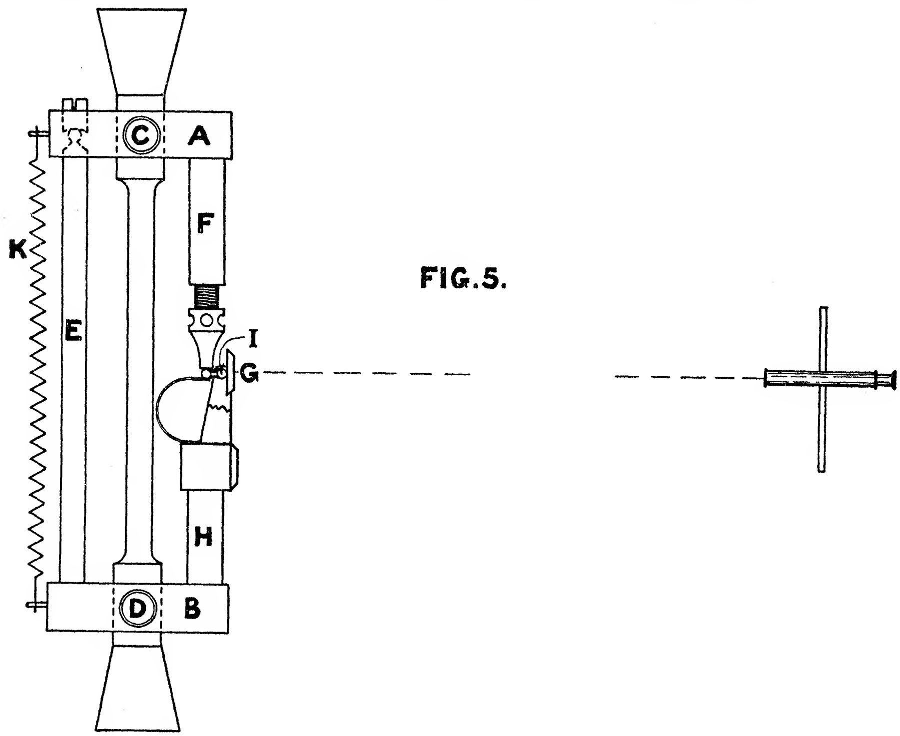


FIG. 5.

spring K. The pillar F, which is fixed to the ring A, engages at its lower end with a steel ball at the end of an arm (about 0·08 inch long) attached to the mirror G. This mirror is pivoted in bearings attached to the pillar H (fixed to ring B) so that it can turn about a horizontal axis I. The reflection of a vertical scale in the mirror is observed in a telescope at a distance of about 6 feet. It is possible to read the scale correct to 1/10 mm., which corresponds to a change of length of about 1·2 millionths of an inch. A fixed mirror carried on the piece serves to measure any tilt of the apparatus as a whole. The instrument is calibrated with sufficient accuracy for the purpose by loading the piece with a known tension.

Change of temperature in the piece, due either to conduction of heat to or from the testing machine attachments, or to slow change in the room temperature, causes a gradual change of length. This was allowed for by observing the rate of change before and after an observation.

It was found that after putting the piece in the testing machine (having previously compressed it in the compression machine) the zero was at first not constant; that is to say, after the application and removal of a load of, say, 2 tons per square inch, whose effects as regards elastic hysteresis must be quite negligible, the reading did not return to the same value. After a few applications and removals of this small load, however, the instrument seemed to settle down and the full tension load was then put on and removed, the corresponding change of length being noted.

In order to make quite certain that the change of length observed as the result of application and removal of tension in a piece which had previously been compressed was really due to the elastic properties of the material, and not in any way to the extensometer, a control experiment was performed. The piece was put in the tension machine, and a load equivalent to 10 tons per square inch was applied and removed a number of times. The piece was then removed from the machine, and handled as nearly as possible in the same way as after its removal from the compression machine in the experiments which have been described. It was then replaced in the tension machine and treated again in the same way, that is, a load of about 2 tons per square inch was applied and removed about a dozen times, so that the zero became perfectly constant. The full tension load of 10 tons per square inch was then applied and removed, and the consequent change of length was noted. It will be seen that in this control experiment every circumstance is exactly the same, except that, prior to its commencement, the piece was loaded in tension instead of in compression, and was brought into the cyclical state corresponding to the application and removal of 10 tons per square inch tension. It was found that the apparent change of length in the control was usually unmeasureable and was never more than about 10 per cent. of the change occurring in the full cycle from compression to tension. It may be inferred from this, and from the general agreement between the experiments, that the width of the hysteresis loop over a range of 20 tons per square inch has certainly been measured correct to within 10 per cent., or, say, to within 1/100 of a ton per square inch.

It should be added that it was found necessary, for the prevention of buckling, to enclose the test-piece when undergoing compression in a closely fitting jacket. This jacket was formed by casting type-metal round the piece, the casting being split so that it could be removed. The jacket was,

of course, taken off when the piece was removed from the compression machine and put into the tension machine. The amount of compressive stress applied was measured with sufficient accuracy for the purpose by means of the extensometer. In this way the effect of friction between the jacket and the piece was eliminated from the measurement of stress.

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*The Direct Production of Characteristic Röntgen Radiations by Cathode Particles.**

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(Communicated by Sir J. J. Thomson, O.M., F.R.S. Received October 29,—Read December 5, 1912.)

It is now well known that many elements can be stimulated to produce characteristic X-rays. So far, the only successful method of obtaining the characteristic rays has been to place the element in the path of a beam of X-rays, whereupon it becomes a secondary radiator; and, if the exciting X-rays have the necessary penetrating power, the characteristic rays will make their appearance.

Some years ago a remarkable paper† by Kaye appeared, in which he showed that if an element, say, copper, were made the anticathode in an X-ray bulb, it could become a source of intense radiation characteristic of copper.

An explanation of these results was at once suggested by Barkla and Sadler.‡ Their view was that the cathode rays on striking the copper plate gave rise to X-rays, some of which penetrated into the copper and thus excited the radiation characteristic of copper. In other words, the effect was *indirectly* produced by the cathode rays.

But with our present knowledge of the amount of energy transformed in such an operation, we can calculate the magnitude of the effect to be expected on this theory. But the amount of characteristic radiation found by Kaye is about ten times greater than can be accounted for by such a calculation.

In the present paper an attempt has been made to find the method of

* The expenses of this research have been partially covered by a Government Grant made through the Royal Society.

† Kaye, 'Phil. Trans.,' A, vol. 209, pp. 123—151.

‡ Barkla and Sadler, 'Phil. Mag.,' May, 1909, pp. 739—760.